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MHD flow of a visco-elastic fluid through porous medium

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Abstract *Here an attempt has been made to study the MHD flow of a dusty, electrically conducting, visco-elastic Rivlin-Ericksen fluid starting from the rest with time-dependent types applied at the free surface. The analytical expression for velocity profiles of the fluid and dust particles have been found by using the Laplace transform technique. Finally the effects of magnetic field time, elastic parameter and mass concentration of dust particles are discussed with the help of graphs and tables.*

Introduction

The study of motions of fluids is one of the most successful and useful applications of mathematics. In classical viscous fluid we know that the fluid exerts a viscosity effect when there is a tendency for shear flow or tangential flow of the fluid. The model of such fluid has been considered to a wider extent. Various types of basic problems of a diversified nature have been solved in this branch. There are other types of fluids called visco-elastic fluids, which possess a certain degree of elasticity in addition to their viscosity. These visco-elastic fluids in the course of their motion store up energy in the material as strain energy. The remaining energy is lost due to viscous dissipation. Clearly, for this class of fluids one cannot neglect the strain, however small it may be. It is responsible for recovery to the original state and thereby reverse flow may occur for removal of the stress. In the process of flow of the fluid the natural state of the fluid changes constantly and tries to attain an instantaneous deformed state. In fact it is never attained completely. The measure of elasticity is, in fact, the lag of the fluid. It is sometimes termed the “memory” of the fluid. A good number of constitutive equations have been proposed at different times for different types of visco-elastic fluids. This has been exhibited in the work of Kapur *et al.* (1982) as well as Bhatnagar (1967).

In recent years, interest in the problem of the flow of dusty visco-elastic fluids has increased enormously through its great importance in the technological field such as the petroleum industry, environmental pollution, purification of crude oils, fluidization, soil dispension by natural wind, and so on. Naturally studies of phase systems are mathematically interesting and

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physically useful for various good reasons. A good number of research workers are engaged in this field of studies including Saffman (1962). Ghosh and Debnath (1986) have investigated hydromagnetic Stokes flow in rotating fluid with suspended small particles. The problem of MHD flow of a dusty visco-elastic conducting fluid through a porous medium between two oscillating plates has been dealt with by Ghosh and Samad (1999) Ghosh *et al.* (2000) studied the hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates.

The main object of this paper is to conduct an investigation of the motion of a hydromagnetic flow of a dusty visco-elastic (Rivlin-Ericksen) fluid starting from rest with time-dependent tangential stress of different types applied at the free surface. The analytical expression for velocity profiles of the fluid and dust particles has been found by using the Laplace transform technique.

Finally, the effects of magnetic field, time, elastic parameter and mass concentration of dust particles are discussed with the help of graphs and tables.

In investigation of the titled problem some related papers (Rivlin and Ericksen, 955; Das, 1988; Debnath and Ghosh, 1988) and books (Batchelor, 1967; Carslaw and Jaeger, 1988; Debnath, 1995) have been consulted.

Basic equation and formulation of the problem

The constitutive equation of Rivlin-Ericksen fluid is given by

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij}$$

$$\tau'_{ij} = \phi_1 A^{(1)}_{ij} + \phi_2 A^{(2)}_{ij} + \phi_3 A^{(1)}_{iK} A^{(1)}_{Kj}$$

where

$$A^{(1)}_{ij} = (u_{i,j} + u_{j,i}) \text{ is the deformation rate tensor,}$$

$$A^{(2)}_{ij} = (a_{i,j} + a_{j,i} + 2u_{i,m}u_{m,j}) \text{ is the visco-elastic tensor,}$$

$$a_i = \frac{\partial u_i}{\partial t} + u_j u_{i,j} \text{ is the acceleration vector,}$$

τ_{ij} is the stress vector, τ'_{ij} is the deviatoric stress tensor which depends on the gradient velocity, acceleration and higher time derivation of velocity, ϕ_1, ϕ_2, ϕ_3 are the co-efficient of viscosity, visco-elasticity and cross-viscosity respectively and are in general functions of temperature and material properties, u_i velocity components.

We now consider the flow of a dusty hydromagnetic visco-elastic Rivlin-Ericksen fluid having a finite thickness h over a plate with time dependent tangential surface, traction which is applied at the upper surface. We take the *y*-axis along the plate in the direction of the flow and *x*-axis being perpendicular to the plate. A uniform magnetic field is applied perpendicular to the plate (see Figure 1).

Using the above condition and results, the governing equation of the problem is governed by the following two equations (Saffman, 1962).

$$\frac{\partial u}{\partial t} = \left(\alpha + \beta \frac{\partial}{\partial t}\right) \frac{\partial^2 u}{\partial x^2} + \frac{K_0 N_0}{\rho}(v - u) - \frac{\alpha}{K} u - \frac{\sigma B_0^2 u}{\rho} \tag{1}$$

and

$$m \frac{\partial v}{\partial t} = K_0(u - v) \tag{2}$$

where u is the fluid velocity, v is the velocity of the dust particles. N_0 the, number of density of the particles, K_0 the Stokes resistance coefficient (= $6\pi\mu r$ for spherical particles of radius r), α the kinematic co-efficient of viscosity, β the kinematic co-efficient of visco-elasticity, B_0 the intensity of the imposed magnetic field, m the mass per unit volume of the dust particles, σ the electrical conductivity of the fluid K and the permeability of the porous medium.

The initial condition is $u = v = 0$ for $t \leq 0$ and $\forall$ x and the boundary conditions are $u = v = 0$ when $x = 0$, $t \geq 0$ and $\left(\alpha + \beta \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial x} = f(t)$, when $x = h$, $t > 0$, where f(t) is the applied surface traction at the upper surface. Introducing the non-dimensional quantities:

$$x' = \frac{x}{h}, t' = \frac{\alpha t}{h^2}, u' = \frac{uh}{\alpha}, v' = \frac{vh}{\alpha}, k' = \frac{k}{h^2}$$

and, omitting dashes over the primed quantities, we get from equations (1) and (2) respectively

$$\frac{\partial u}{\partial t} = \left(1 + E \frac{\partial}{\partial t}\right) \frac{{}^2 u}{\partial x^2} + \lambda(v - u) - Gu \tag{3}$$

and

$$\frac{\partial v}{\partial t} = L(u - v) \tag{4}$$

where

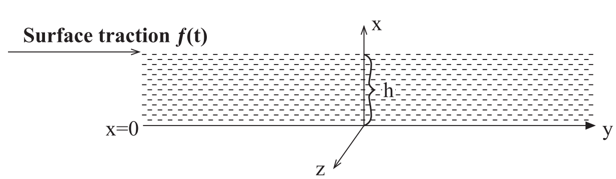

Figure 1.
Flow configuration

$$L = \frac{h^2}{\tau a}, f = \frac{mN_0}{\rho} \text{(mass concentration of the dust particles)},$$

$$\tau = \frac{m}{k_0} \text{(time relaxation of the dust particles)},$$

$$E = \frac{\beta}{h^2} \text{ (elastic parameter)},$$

$$M = B_0 h \sqrt{\frac{\sigma}{\rho\alpha}} \text{ (Hartmann number)},$$

$$G = M^2 + \frac{1}{K}$$

and $\lambda = fL$

The non-dimensional initial condition is

$$u = v = 0 \text{ when } t \leq 0 \text{ for all } x \text{ in } [0,1] \tag{5}$$

and the boundary conditions are

$$u = v = 0 \text{ when } x = 0 \text{ for all } t$$

and

$$\left(1 + E\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} = f(t) \text{ for } x = 1 \tag{6}$$

where

$$f(t) = \frac{h^2}{\alpha^2 \rho} f\left(t' \frac{h^2}{\alpha}\right)$$

Solution of the problem

To solve equations (3) and (4) we use Laplace transform

$$\bar{u}(x,s) = \int_0^{\alpha} u(x,s)e^{-st}dt, \quad Re(s) > 0$$

With the help of initial condition and Laplace transform we get from equations (3) and (4):

$$\frac{d^2\bar{u}}{dx^2} - Q^2(s)u = 0 \tag{7}$$

and

$$\bar{v} = \frac{L\bar{u}}{L+s} \tag{8}$$

where

$$Q^2(s) = \frac{s^2 + s(L + \lambda + G) + GL}{(L + s)(1 + sE)}$$

After using the Laplace transform the boundary conditions become

$$\bar{u} = \bar{v} = 0 \text{ at } x = 0 \tag{9}$$

and

$$(1 + Es)\frac{d\bar{u}}{dx} = \bar{f}(s) \quad \text{at } x = 1 \tag{10}$$

where

$$f(s) = \int_0^\infty f(t)e^{-st}dt$$

The solution of equation (7) with the boundary conditions (9) and (10) is given by

$$\bar{u} = \frac{\bar{f}(s) \sin h\ Qx}{Q(s)(1 + sE) \cos hQ} \tag{11}$$

$$\bar{v} = \frac{L\bar{f}(s) \sin h\ Qx}{Q(s)(s + L)(1 + sE) \cos h\ Q} \tag{12}$$

Case I: Flow due to periodic surface traction

For periodic surface traction, we take f(t) = a cosωt; a and ω are constant and as such

$$f(s) = \frac{as}{s^2 + \omega^2}.$$

Putting the value of $\bar{f}(s)$ in equations (11) and (12) and then inverting the transform by the method of calculus of residues, we get the velocity components of the fluid and dust particles, which are respectively given by

$$\frac{\mu}{\alpha} = A_1(y) \cos \omega t + B_1(y) \sin \omega t + 2\sum_0^\infty (-1)^n \sin(2n + 1)\frac{\pi y}{2} \times$$

$$\left[\frac{\Delta_n^{(1)} s_n^{(1)} e^{s_n^{(1)}t}}{\left(s_n^{(1)^2} + \omega^2\right)} + \frac{\Delta_n^{(2)} s_n^{(2)} e^{s_n^{(2)}t}}{\left(s_n^{(2)^2} + \omega^2\right)}\right] \tag{13}$$

and

$$\frac{v}{\alpha} = \frac{A_1(y)(L^2 - L\omega)\cos\omega t}{(L^2+\omega^2)} + \frac{B_1(y)(L^2 + L\omega)\sin\omega t}{(L^2+\omega^2)}$$

$$+ 2L\sum_{n=0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi y}{2}\times$$

$$\left[\frac{\Delta_n^{(1)} s_n^{(1)} e^{s_n^{(1)} t}}{\left(s_n^{(1)^2} + \omega^2\right)\left(s_n^{(1)} + L\right)} + \frac{\Delta_n^{(2)} s_n^{(2)} e^{s_n^{(2)} t}}{\left(s_n^{(2)^2} + \omega^2\right)\left(s_n^{(2)} + L\right)}\right] \quad (14)$$

where

$$A = \frac{(GL-\omega^2)(L-E\omega^2) + (\lambda+G+L)(1+LE)}{(L-E\omega^2)^2 + (1+LE)^2\omega^2},$$

$$B = \frac{\omega[(\lambda+G+L)(L-E\omega^2) + (GL-\omega^2)(1+LE)]}{(L-E\omega^2)^2 + (1+LE)^2\omega^2},$$

$(\mu_1, \mu_2) = \left(\frac{\sqrt{A^2+B^2} \pm A}{2}\right)^{\frac{1}{2}}$, $s_n^{(1)}$ and $s_n^{(2)}$ are the roots of the equation

$$(1+Ep_n^2)s^2 + [\lambda+G+L+(1+LE)p_n^2]s + L(G+p_n^2) = 0,$$

$$p_n = (2n+1)\frac{\pi}{2}, n = 0, 1, 2, \ldots$$

$$\Delta_n^{(j)} = \frac{(1+s_n^{(j)}E)(s_n^{(j)}+L)^2}{[1-E(\lambda+G)]s_n^{(j)^2} - 2L(GE-1)s_n^{(j)} + L(\lambda+L-GLE)} j = 1, 2$$

$$K_1 = \cos h\mu_1 \cos\mu_2, \quad K_2 = \sin h\mu_1 \sin\mu_2, \quad K_3 = K_1^2 + K_2^2$$

$$\xi(y) + K_1 \sin h\ \mu_1 y \cos\ \mu_2 y + K_2\ \cos h\mu_1 \sin\ \mu_2 y$$

$$\eta(y) = K_1 \cosh\mu_1 y \sin\mu_2 y - K_2 \sinh\mu_1 y \cos\mu_2 y$$

$$A_1(y) = \frac{(\mu_1 - E\omega\mu_2)\xi(y) + (E\omega\mu_1 + \mu_2)\eta(y)}{K_3[(\mu_1 - E\omega\mu_2)^2 + (E\omega\mu_1 + \mu_2)^2]}$$

$$B_1(y) = \frac{(E\omega\mu_1 + \mu_2)\xi(y) - (\mu_1 - E\omega\mu_2)\eta(y)}{K_3[(\mu_1 - E\omega\mu_2)^2 + (E\omega\mu_1 + \mu_2)^2]}$$

If we take $t \rightarrow \propto, \omega \rightarrow 0$ in equations (12) and (13), we get the velocity distributions for constant shearing stress in steady state. In this case, it is very interesting to note that the fluid and dust particles move with the same velocity of magnitude

$$u = \frac{\sinh \sqrt{G}x}{\sqrt{G}\cosh \sqrt{G}}$$

which is independent of elastic parameter (E) but depends on Hartmann number. Further to this, if we take $L \rightarrow \propto$ in equation (14), we get the velocity of clean fluid, which is same as equation (12).

Shear stress at the base

The shear stress at the lower plate is given by

$$\tau_1 = \left[\rho\left(\alpha + \beta\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}\right]_{x=0} \tag{15}$$

Introducing non-dimensional quantities:

$$x' = \frac{x}{h}, \quad u' = \frac{uh}{\alpha}, \quad t' = \frac{\alpha t}{h^2}, \quad \tau_1' = \frac{\tau_1 h^2}{\rho\alpha^2}$$

and, omitting dashes over the primed quantities, we have from equation (15)

$$\tau_1 = \left[\left(1 + E\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}\right] \tag{16}$$

where

$$E = \frac{\beta}{h^2} \text{ is the elastic parameter}$$

From equations (12) and (16) we have

$$\frac{\tau_1}{a} = \frac{(K_1 \cos\omega t + K_2 \sin\omega t)}{K_3} + \pi \sum_{n=0}^{\infty} (-1)^n (2n+1)$$

$$\left[\frac{\Delta_n^{(1)} s_n^{(1)} (1 + E s_n^{(1)}) e^{s_n^{(1)} t}}{\left(s_n^{(1)^2} + \omega^2 \right)} + \frac{\Delta_n^{(2)} s_n^{(2)} (1 + E s_n^{(2)}) e^{s_n^{(2)} t}}{\left(s_n^{(2)^2} + \omega^2 \right)} + \right]$$

which gives the shear stress at the plate.

Case II: Flow due to impulsive shearing stress

In the present circumstances, we take $f(t) = S\delta(t)$, where S is a constraint and $\delta(t)$ is Dirac delta function.

From equations (11) and (8), we get

$$\bar{u} = \frac{S \sinh\ py}{p(1 + sE) \cosh p} \text{ and } \bar{v} = \frac{S \sinh py}{p(1 + sE)(s + 1) \cosh p}$$

By inverse Laplace transform, we obtain from $\bar{u}$ and $\bar{v}$ the expression for u and v as

$$\frac{u}{S} = 2 \sum_{n=0}^{\infty} (-1)^n \sin(2n+1) \frac{\pi y}{2} \left[\Delta_n^{(1)} e^{s_n^{(1)} t} + \Delta_n^{(2)} e^{s_n^{(2)} t} \right] \quad (17)$$

and

$$\frac{v}{S} = 2 \sum_{n=0}^{\infty} (-1)^n \sin(2n+1) \frac{\pi y}{2} \left[\Delta_n^{(1)} e^{s_n^{(1)} t} + \Delta_n^{(2)} e^{s_n^{(2)} t} \right] \quad (18)$$

The shear stress at the fixed base (as in the previous case) is

$$\frac{\tau_1}{s} = \Pi \sum_{n=0}^{\infty} (-1)^n (2n+1) \left[\Delta_n^{(1)} (1 + E s_n^{(1)}) e^{s_n^{(1)} t} + \Delta_n^{(2)} (1 + E s_n^{(2)}) e^{n^{(2)} t} \right] \quad (19)$$

Case III: Flow due to constant shearing stress acting for a finite time T

In this case, let us take

$$f(t) = S[H(t) - H(t - T)],$$

where S is a constant and H(t) is the Heaviside unit function defined by

$$\begin{aligned} H(t) &= 0 \quad \text{for } t < 0 \\ &= 1 \quad \text{for } t > 0 \end{aligned}$$

Then from equations (8) and (11)

$$\bar{u} = \frac{S(1 - e^{-sT}) \sinh px}{s(1 + sE)p \cosh p} \tag{20}$$

and

$$\bar{v} = \frac{S(1 - e^{-sT}) \sinh px}{s(1 + sE)(1 + \tau s)p \cosh p} \tag{21}$$

By inverse Laplace transform, we have

$$\frac{u}{S} = \frac{\sinh \sqrt{G}y}{\sqrt{G} \cosh \sqrt{G}} + 2\sum_{n=0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi x}{2}\left[\Delta_n^{(1)} e^{s_n^{(1)}t} + \Delta_n^{(2)} e^{s_n^{(2)}t}\right] \quad \text{when } 0 < t \leq T \tag{22}$$

$$= 2\sum_{n=0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi y}{2}\left[\Delta_n^{(1)}\left(1 - e^{-s_n^{(1)}T}\right)e^{s_n^{(1)}t} + \Delta_n^{(2)}\left(1 - e^{-s_n^{(2)}T}\right)e^{s_n^{(2)}t}\right] \quad \text{when } t > T \tag{23}$$

and

$$\frac{v}{S} = \frac{\sinh \sqrt{G}y}{\sqrt{G} \cosh \sqrt{G}} + 2L\sum_{n=0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi x}{2}\left[\frac{\Delta_n^{(1)} e^{s_n^{(1)}t}}{s_n^{(1)} + L} + \frac{\Delta_n^{(2)} e^{s_n^{(2)}t}}{s_n^{(2)} + L}\right] \quad \text{when } 0 < t \leq T \tag{24}$$

$$= 2L\sum_{n=0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi x}{2}\left[\frac{\Delta_n^{(1)}\left(1 - e^{-s_n^{(1)}T}\right)e^{s_n^{(1)}t}}{s_n^{(1)} + L} + \frac{\Delta_n^{(2)}\left(1 - e^{-s_n^{(2)}T}\right)e^{s_n^{(2)}t}}{s_n^{(2)} + L}\right] \tag{25}$$

when $t > T$

Discontinuity in the velocity

We observe that, when the shear stress is withdrawn, there is a jump discontinuity of amount

$$J_{\frac{u}{s}} = \frac{\sinh\sqrt{G}x}{\sqrt{G}\cosh\sqrt{G}} + 2\sum_{0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi x}{2}\left(\Delta_n^{(1)} + \Delta_n^{(2)}\right) \quad (26)$$

and

$$J_{\frac{v}{s}} = \frac{\sinh\sqrt{G}x}{\sqrt{G}\cosh\sqrt{G}} + 2L\sum_{0}^{\infty}(-1)^n \sin(2n+1)\frac{\pi x}{2}\left(\frac{\Delta_n^{(1)}}{s_n^{(1)}+L} + \frac{\Delta_n^{(2)}}{s_n^{(2)}+L}\right) \quad (27)$$

in the velocity of the fluid and the dust particles respectively.

Shear stress at the fixed base

The shear stress at the fixed base (as in Case I) is

$$\frac{\tau_1}{S} = \sec h\sqrt{G} + \Pi\sum_{n=0}^{\infty}(-1)^n(2n+1) \left[\Delta_n^{(1)}(1+Es_n^{(1)})e^{s_n^{(1)}t} + \Delta_2^{(2)}(1+Es_n^{(2)})e^{s_n^{(2)}t}\right] \quad \text{when } 0 < t \le T \quad (28)$$

$$= \Pi\sum_{n=0}^{\infty}(-1)^n(2n+1)\left[\Delta_n^{(1)}\left(1+Es_n^{(1)}\right)\left(1-e^{-s_n^{(1)}T}\right)e^{-s_n^{(1)}t} + \Delta_n^{(2)}\left(1+Es_n^{(2)}\right)\left(1-e^{-s_n^{(2)}T}\right)e^{-s_n^{(2)}t}\right] \quad \text{when } t > T \text{ (time)} \quad (29)$$

Discontinuity in the shear stress

When the surface traction is withdrawn, at time t = T there is a jump discontinuity of amount

$$J_{T_s} = \sec h\sqrt{G} + \Pi\sum_{n=0}^{\infty}(-1)^n(2n+1) \left[\Delta_n^{(1)}(1+Es_n^{(1)}s_n^{(1)}) + \Delta_n^{(2)}(1+E_n^{(2)}s_n^{(2)})\right] \quad (30)$$

which is independent of time t.

Numerical discussion

In the graphs and tables the displacement profiles u/a and v/a for Case I have been exhibited against the space co-ordinates x with the assumed values of E, the elastic parameter, the values of M, the Hartmann number, the values of f, the mass concentration of the dust particles as well as time parameter t. In the same fashion the profiles of the skin-friction τ_1 have been exhibited against the time t, assuming various values of the elastic parameter E, the values of G and the values of f.

Moreover, in the graphs and tables of the velocity distribution and the skin-friction the relevant quantities necessary for exhibition of the variations have been prepared for ready observation and necessary action.

Finally, the observation of the graphs and the corresponding analysis of the variations, the characteristics features and truth emerging have been presented in the discussion in the last article.

Discussion and conclusion

It is very clear from the numerical computation of the velocity profile of the fluid, as exhibited in Figure 3, that the velocity of the fluid is diminished due to the presence of the transverse magnetic field. Whenever the elastic parameter (E) increases, the velocity of the fluid diminishes, as in Figure 2. Clearly from Figure 4, as t increases, the value of the velocity component of the fluid gradually increases.

From Tables I, II and III the same phenomenon is found in the case of the velocity of dust particles. Further to this, it is also obvious from Tables IV and V that the velocity components of the fluid element and dust particles increases with the increase of mass concentration of the dust particles (f). Also from Figures 5 and 6 we see that the shear stress at the fixed base is periodic with

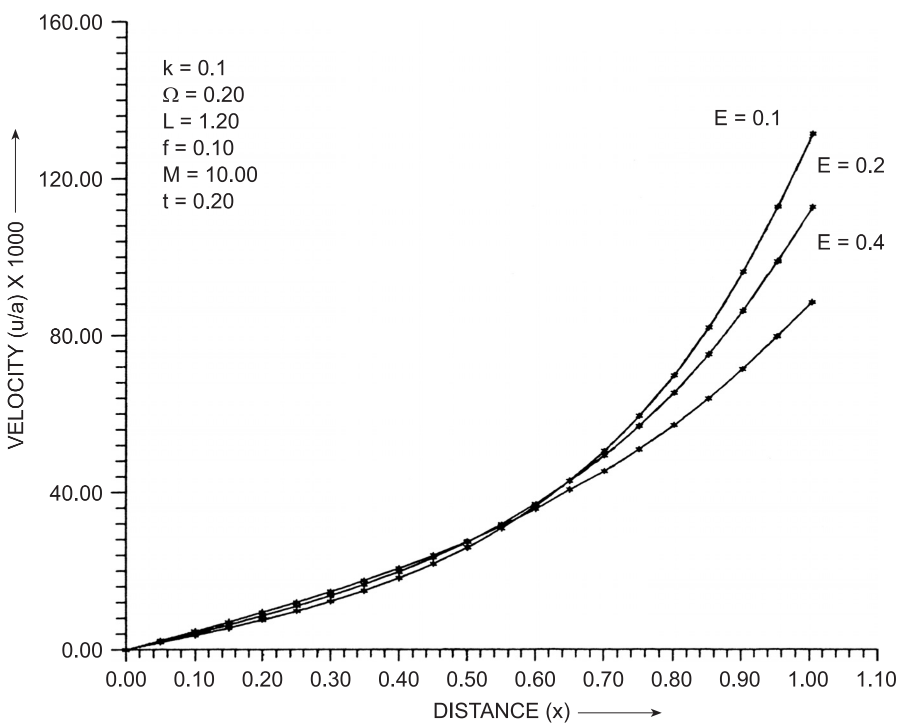

Figure 2.
Velocity profiles for different values of E

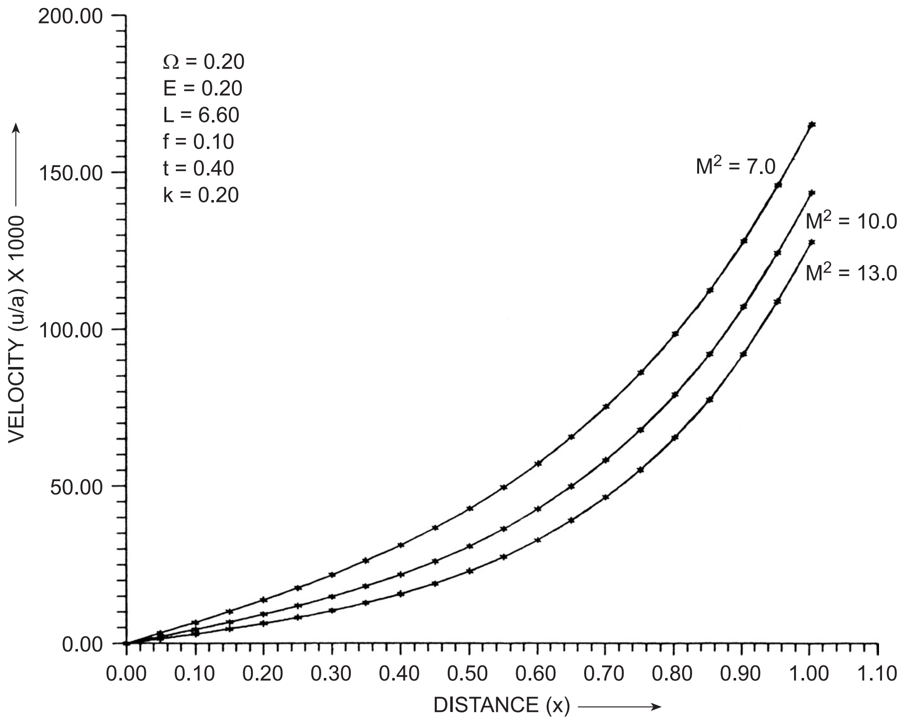

Figure 3.
Velocity profiles for different values of M

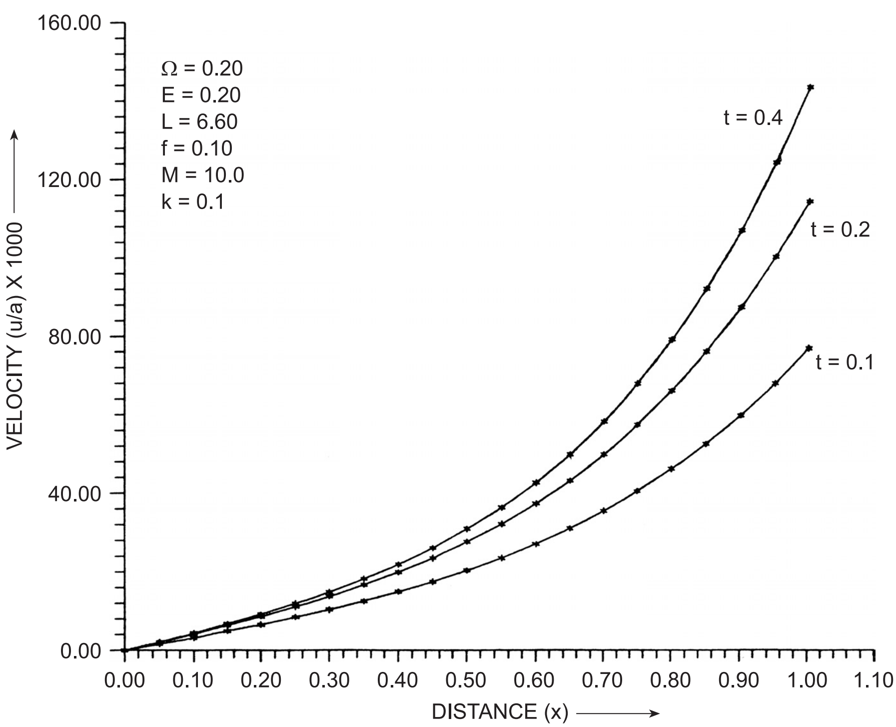

Figure 4.
Velocity profiles for different values of t

x	E 0.1	0.2	0.4
0.050	0.000215	0.000257	0.000273
0.100	0.000435	0.000519	0.000549
0.150	0.000666	0.000789	0.000829
0.200	0.000915	0.001072	0.001118
0.250	0.001186	0.001372	0.001417
0.300	0.001487	0.001696	0.001730
0.350	0.001825	0.002047	0.002059
0.400	0.002211	0.002432	0.002408
0.450	0.002650	0.002856	0.002779
0.500	0.003158	0.003329	0.003177
0.550	0.003743	0.003854	0.003604
0.600	0.004425	0.004444	0.004066
0.650	0.005216	0.005105	0.004565
0.700	0.006141	0.005852	0.005109
0.750	0.007217	0.006693	0.005700
0.800	0.008479	0.007645	0.006345
0.850	0.009951	0.008720	0.007048
0.900	0.011677	0.009939	0.007819
0.950	0.013694	0.011319	0.008663
1.000	0.015955	0.012818	0.009549

Table I. Velocity profile of dust particles against x for different values of E when ω = 0.20, L = 6.67, f = 0.10, t = 0.20, M = 3.16, k = 0.1

x	t 0.1	0.2	0.4
0.050	0.000075	0.000177	0.000287
0.100	0.000151	0.000357	0.000580
0.150	0.000229	0.000543	0.000885
0.200	0.000310	0.000739	0.001209
0.250	0.000397	0.000947	0.001557
0.300	0.000489	0.001171	0.001939
0.350	0.000588	0.001415	0.002359
0.400	0.000696	0.001684	0.002830
0.450	0.000814	0.001981	0.003358
0.500	0.000945	0.002312	0.003957
0.550	0.001089	0.002687	0.004636
0.600	0.001250	0.003099	0.005413
0.650	0.001429	0.003567	0.006298
0.700	0.001630	0.004097	0.007316
0.750	0.001854	0.004695	0.008480
0.800	0.002107	0.005375	0.009822
0.850	0.002390	0.006143	0.011360
0.900	0.002708	0.007018	0.013133
0.950	0.003067	0.008010	0.015171
1.000	0.003454	0.009090	0.017417

Table II. Velocity profile of dust particles against χ for different values of t when ω = 0.20, L = 6.67, f = 0.1, E = 0.1 M = 3.16, K = 0.1

x	M 7	10	13
0.050	0.000404	0.000206	0.000287
0.100	0.000816	0.000419	0.000580
0.150	0.001229	0.000641	0.000885
0.200	0.001683	0.000882	0.001209
0.250	0.002153	0.001144	0.001577
0.300	0.002658	0.001438	0.001939
0.350	0.003204	0.001768	0.002359
0.400	0.003803	0.002146	0.002830
0.450	0.004460	0.002578	0.003358
0.500	0.005191	0.003079	0.003957
0.550	0.006000	0.003657	0.004636
0.600	0.006909	0.004332	0.005413
0.650	0.007923	0.005115	0.006298
0.700	0.009067	0.006031	0.007316
0.750	0.010351	0.007098	0.008480
0.800	0.011802	0.003848	0.009820
0.850	0.013437	0.009805	0.011360
0.900	0.015288	0.011511	0.013133
0.950	0.017378	0.013503	0.015171
1.000	0.019643	0.015732	0.017417

Table III. Velocity profile of dust particles against x for different values of M when $\omega = 0.20$, $L = 6.67$, $f = 0.1$, $E = 0.2$, $t = 0.40$, $k = 0.1$

x	λ 0.1	0.2	0.3
0.050	0.000259	0.000258	0.000256
0.100	0.000523	0.000520	0.000516
0.150	0.000795	0.000790	0.000784
0.200	0.001081	0.001074	0.001067
0.250	0.001383	0.001374	0.001366
0.300	0.001710	0.001699	0.001688
0.350	0.002063	0.002050	0.002038
0.400	0.002451	0.002437	0.002423
0.450	0.002878	0.002862	0.002846
0.500	0.003355	0.003336	0.003318
0.550	0.003884	0.003864	0.003844
0.600	0.004479	0.004457	0.004435
0.650	0.005145	0.005121	0.005098
0.700	0.005897	0.005872	0.005847
0.750	0.006744	0.006717	0.006690
0.800	0.007704	0.007675	0.007647
0.850	0.008786	0.008756	0.008727
0.900	0.010015	0.009984	0.009953
0.950	0.011405	0.011373	0.011342
1.000	0.012915	0.012883	0.012851

Table IV. Velocity profile of dust particles against χ for different values of f when $\omega = 0.20$, $L = 6.67$, $E = 0.2$, $t = 0.40$, $M = 3.16$, $k = 0.1$

Table V.
Velocity profile of fluid for different values of f when ω = 0.20, L = 0.15, E = 0.20, t = 0.40, M = 3.16, k = 0.1

x	λ 0.1	0.2	0.3
0.05	0.002076	0.002056	0.002036
0.1	0.004194	0.004153	0.004113
0.15	0.006385	0.006324	0.006263
0.2	0.008701	0.008619	0.008538
0.25	0.011171	0.011069	0.010967
0.3	0.013859	0.013735	0.013612
0.35	0.016793	0.016647	0.016503
0.4	0.020051	0.019883	0.019717
0.45	0.023666	0.023476	0.023288
0.5	0.027735	0.027523	0.027312
0.55	0.032299	0.032064	0.031831
0.6	0.037477	0.03722	0.036964
0.65	0.043325	0.043045	0.042768
0.7	0.04999	0.049689	0.04939
0.75	0.057551	0.057229	0.05691
0.8	0.066187	0.065846	0.065509
0.85	0.076011	0.075653	0.075299
0.9	0.087242	0.086872	0.086504
0.95	0.100045	0.099665	0.099289
1	0.114046	0.113664	0.113284

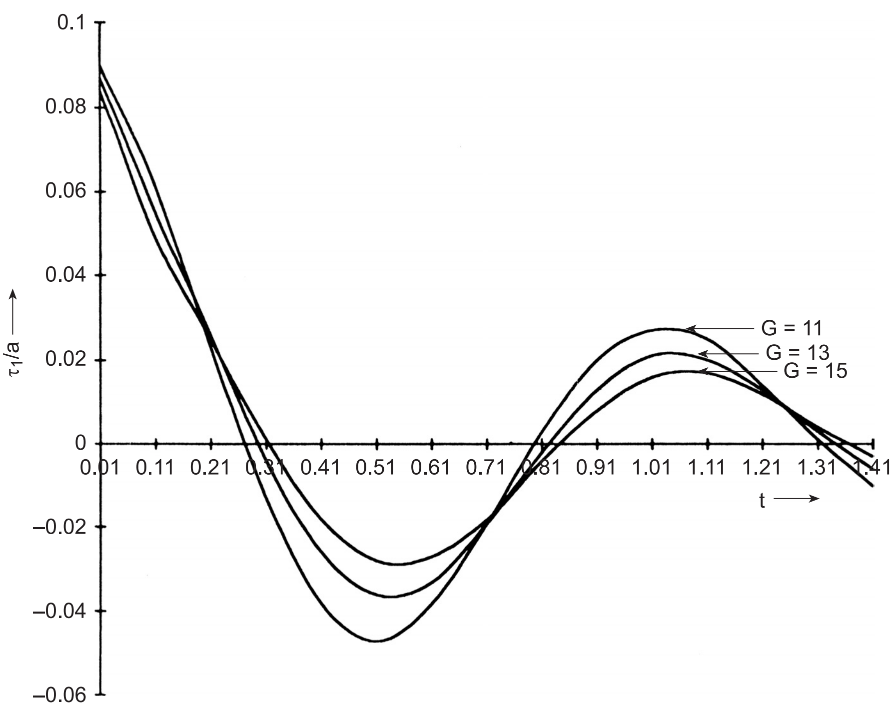

Figure 5.
Shear stress profiles for different values of G when E = 0.1, f = 0.1, L = 6.67, ω = 6

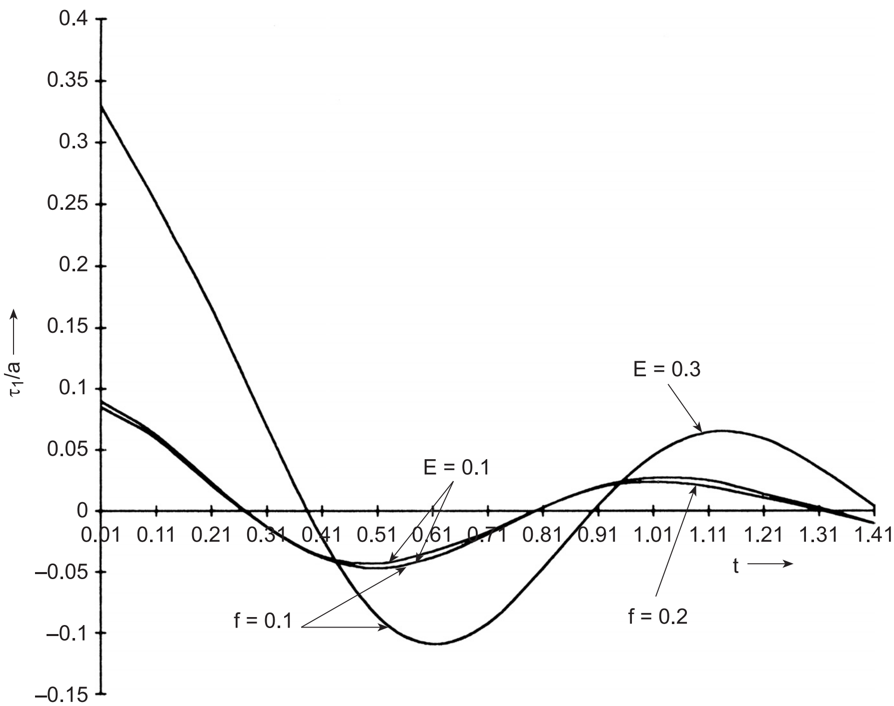

Figure 6.
Shear stress profiles for different values of E and f when M = 1, K = 0.1, L = 6.67, ω = 6

respect to time. So we conclude that (i) for fixed K, the amplitude of the shear stress decreases with the increase of Hartmann number (M) and (ii) for fixed M the amplitude of the shear stress decreases with the decrease of the porosity of the medium (K). Figure 6 reveals that the amplitude of the shear stress increases with the increase of the elastic parameter (E) and slightly decreases with the increase of f, the mass concentration of the dust particles.

From the above discussion, we immediately come to the conclusion that the velocity fields of the fluid and dust particles decrease with the increase of the Hartmann number (M), that is, the magnetic field decelerates the flow. Thus the magnetic number characterising the intensity of the magnetic field always arrests the velocity of dusty fluid. Similarly, if the elastic parameter of the fluid increases, the velocity of the fluid element diminishes.

Both these phenomena are in conformity with the concept that the magnetic lines of force always drag the fluid perpendicular to them. Magnetic lines of force are as if frozen in the fluid.

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